Modeling Design for a Matrix Converter with a Generator as Input

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Abstract - This paper proposes a model and design based on the behavior of a matrix converter and a generator. Presently, applications for grid connection have been considered widely such as micro-gas turbines, wind power systems, engine generators and so on. However, a very large impedance power supply such as the generator has not been sufficiently examined. Especially the PM generator which is used in various applications has large synchronous reactance. To perform a simple characteristic analysis, an ideal mathematical model excluding switching parts is introduced. An input filter characteristic is discussed by using an ideal mathematical model. Thus, the cause of the unstable behavior is clarified. In order to verify about the stability operation, a proposed stability control on the rotating frame is applied to the generator. The proposed method can control the generator current and the output voltage regardless of the generator speed. In addition, a control of optimal input power factor is proposed to obtain maximum modulation index of the matrix converter. Moreover, the fluctuation in the output voltage is able to compensate by the proposed voltage compensation method. This paper confirms the validity of the proposed strategy by simulation and experiment. The large oscillation on the generator terminal voltage and current are suppressed by the proposed stability control. Moreover, the proposed control achieves stability at motor load 1.5kW and 3.7\% of the input current THD.

I. INTRODUCTION

Recently, matrix converters, that can directly convert an AC power supply voltage into an AC voltage of variable amplitude and frequency without large energy storage, such as electrolytic capacitor, have been actively studied [1-9]. Matrix converters have advantages of size reduction, lightweight, long-life and high efficiency power supplies, compared with the use of a PWM rectifier and inverter system.

However, matrix converters are mainly considered for motor drive system applications, such as elevators and air conditioners. Lately, applications for grid connection have been considered, such as micro-gas turbines, wind power systems, engine generators and so on. The behavior of matrix converters under the large line impedance of a power grid has been investigated [4] [8]. However, very large impedance power supplies, such as generators have not been sufficiently examined.

For example, it has not been sufficiently discussed that the terminal voltage of the generator and input current are unstable due to the resonance between synchronous reactance and the input capacitor. The stability control method of the input current considering the high line impedance has been already proposed [4]. However, that effect has not been reported in case of the generator as input.

For a stable operation, it is important to obtain a system modeling before control it. However, it is difficult to discuss the behavior of the matrix converter because the behavior of the input current interference directly to the output control. In order to analyze the operation of the matrix converter easily, a simple model such as mathematical model is required.

This paper discusses a model for a matrix converter with a high impedance power supply such as a generator. In this paper, the ideal mathematical model for the matrix converter is introduced to discuss the stabilization of the input filter by using synchronous reactance of the generator. In addition, the design of a stability control is sufficiently provided based on the ideal mathematical model. Since, the control method of the generator using the matrix converter has not been discussed, yet. Therefore, the output voltage of the matrix converter is depended on an input power factor because a voltage transfer ratio is limited by the input power factor. Therefore, the input power factor control has to be considered from the point of view of the maximum voltage transfer ratio. Note that, the output voltage of the matrix converter depends on the input terminal voltage. The output voltage should keep to constant for the stable operation. Therefore, this paper also proposes the output voltage compensation method using frequency information of the induced e.m.f. instead of the input terminal voltage of the matrix converter.

This paper provides the control characteristics of the matrix converter based on simulation and experimental results as follows; (1) the fundamental operation with R-L load, (2) the relation among the input current phase, the terminal voltage and the output voltage, (3) the operation of the induction motor load, (4) the acceleration and deceleration characteristics of the induction motor, and (5) the acceleration and deceleration of the generator with an output voltage transfer ratio. The results of the simulation and the experiment confirmed the validity of the proposed system.

II. MATHEMATICAL MODEL OF MATRIX CONVERTER

A. Mathematical model without switching operation

Fig. 1 shows a circuit diagram of the matrix converter. The simulation of the matrix converter including motor or generator consists of control part to obtain the output
voltage command and switching models for the PWM operation. The simulation of these applications requires a very long time due to short time step for the PWM operation. Therefore, a mathematical model is introduced in order to achieve the simulation at a shorter time.

Fig. 2 shows an ideal mathematical model of the matrix converter. The behavior of the input side of the matrix converter is expressed by current sources. Likewise, the behavior of the output side is expressed by voltage sources. An active power of the input side always equals to the converter. The behavior of the input side of the matrix model, where the converter loss can be neglected. Then the relationship between input current source and output voltage is given by (1)

\[ V_{in}I_{in} \cos \phi = V_{out}I_{out} \cos \phi \]  

(1)

where \( V_{in} \) is the input voltage, \( V_{out} \) is the output voltage, \( I_{in} \) is the input current, \( I_{out} \) is the output current, \( \cos \phi \) is the input power factor and \( \cos \phi \) is the output power factor.

However, it is complicated to calculate the instantaneous value of the input power factor and load power factor. Hence, this paper considers the relation between the output and input side on a rest frame known as \( \alpha-\beta \) frame. The output voltage is obtained by multiplying input current command by the input voltage. Likewise, the input current is obtained by multiplying the output voltage command by the load current. Therefore the output voltage and input current are expressed in (2) and (3) considering is active power.

\[
\begin{align*}
    v_{in} & = (v_{in\alpha}+v_{in\beta})v_{in\alpha} + v_{in\beta}v_{in\beta} \\
    v_{in\beta} & = (v_{in\alpha}+v_{in\beta})v_{in\alpha} + v_{in\beta}v_{in\beta} \\
    i_{in\alpha} & = (v_{in\alpha}+v_{in\beta})i_{in\alpha} + v_{in\beta}i_{in\beta} \\
    i_{in\beta} & = (v_{in\alpha}+v_{in\beta})i_{in\alpha} + v_{in\beta}i_{in\beta}
\end{align*}
\]  

(2)

where \( v_{in\alpha} \) \( v_{in\beta} \) \( i_{in\alpha} \) \( i_{in\beta} \) are the input voltage, \( i_{in\alpha} \) \( i_{in\beta} \) are the input current, \( i_{in\alpha} \) \( i_{in\beta} \) are the input current command normalized by the output current, \( v_{in\alpha} \) \( v_{in\beta} \) are the output voltage, \( v_{in\alpha} \) \( v_{in\beta} \) are the output voltage command normalized by the input voltage and \( i_{in\alpha} \) \( i_{in\beta} \) are the output current.

Fig. 3 shows a simulation results using ideal mathematical model with the synchronous reactance and filter capacitor. In case of the generator as input, the synchronous reactance is added to the input part of the mathematical model. In addition, the input filter capacitor is also connected. As a result, the resonance between synchronous reactance and filter capacitor occurs. The input terminal voltage and input current have large oscillation due to the filter resonance. Likewise, the output voltage also has distortion. That is, the synchronous reactance is the cause of unstable.

B. Behavior of the synchronous reactance in a generator

Fig. 4 shows a single phase equivalent circuit of the input filter with synchronous reactance. The output active power must be kept constant even if the input terminal voltage has fluctuation. The relationship among the terminal voltage, the load current and the output power are obtained as (4)

\[ P_{in} + \Delta P_{in} = v_{in\alpha}i_{in\alpha} + \Delta v_{in\alpha}i_{in\alpha} + v_{in\beta}i_{in\beta} + \Delta v_{in\beta}i_{in\beta} \]  

(4)

where \( P_{in} \) \( \Delta P_{in} \) \( v_{in\alpha} \) \( v_{in\beta} \) \( i_{in\alpha} \) \( i_{in\beta} \) are output power, terminal voltage, load current and the subscript “s” is steady term, the symbol “\( \Delta \)” means ripple components.

The product of the ripple components \( \Delta v \) and \( \Delta i \) can be neglected due to small value. Therefore, the steady terms and ripple components of the load current are given by (5) and (6)
\[ i_{ca} = P_{ca} = \frac{1}{R_c} v_{ca} \]  
\[ \Delta i_o = \frac{\Delta P_o - \Delta v_i i_o}{v_{ca}} \]  
(5)

where \( R_c \) is the load resistance equivalent to the constant power load.

In (6), \( \Delta P_o \) equals zero when the output power is controlled to constant. Then, the input current ripple is obtained as (7)

\[ \Delta i_o = -\frac{i_{ca}}{v_{ca}} \Delta v_i = -K_o \Delta v_i \]  
(7)

where \( K_o \) is a feedback gain defined by \( i_{ca} / v_{ca} \).

Fig. 5 shows a block diagram of the input filter with the output power is controlled constantly. \( H_1(s) \) and \( H_2(s) \) represent as a filter to divide the steady term and the ripple component respectively as in fig. 5. For example, each filter has characteristics as low pass filter and high pass filter. In this case, a transfer function of the input filter is given by (8)

\[ \frac{V_f(s)}{V_g(s)} = \frac{1}{L_f C_f s^2 + \frac{1}{C_f} \left( \frac{1}{R_c} - H_1(s) - H_2(s) K_o \right) s + \frac{1}{L_f C_f}} \]  
(8)

It seems that \( K_o \), which is a feed back gain of the ripple component, behaves a negative susceptance. The positive feedback gain \( K_o \) becomes larger when the output current control is high response because the high control gain strongly suppresses the input voltage disturbance. As a result, the system becomes more unstable.

In a matrix converter with power grid as input, the input filter resonance is suppressed by a damping resistor connected to the input filter reactor in parallel. However, for the generator as input, the damping resistor can not be inserted to the synchronous reactance in parallel practically. Therefore, the control for suppressing resonance is required to the input side.

III. CONTROL STRATEGY

A. Input Current Stability Control

To suppress the oscillation of the input side, this paper applies a damping control [4] [10] for the generator. In the damping control, the distortion component of the terminal voltage is removed by using a band pass filter (BPF). After that, the distortion component is subtracted to the input current command. In this case, a band pass frequency depends on the input frequency. Therefore, the band pass frequency has to be adjusted by the output frequency of the generator. On the basis of these problems, this paper achieves the stability control method using an adaptive BPF on a rotating frame known a d-q frame.

Fig. 6 shows a block diagram of the stability control on the d-q frame. The fundamental frequency component on the d-q frame becomes a constant value, i.e. DC signal. In addition, harmonics components appear as a ripple. Therefore, harmonics components can be completely removed by using a low pass filter (LPF) which has a long time constant. The cut-off frequency of the LPF is set to a quick response, faster than the speed response of the generator. This method is acceptable for generator input since the adaptive BPF can be constructed by LPF.

Fig. 7 shows a block diagram of the transfer function of the input current stability control. The transfer function from generator voltage to capacitor voltage is described by (9)

\[ \frac{\Delta V_f(s)}{\Delta V_g(s)} = \frac{1}{L_f C_f s^2 + \frac{1}{C_f} (K_d - K_i) s + \frac{1}{L_f C_f}} \]  
(9)

Then the damping factor \( \zeta \) is calculated by (10)

\[ \zeta = 1 - \frac{1}{2} \sqrt{\frac{C_f}{K_d - K_i}} \]  
(10)

The damping factor \( \zeta \) can be increased by the feedback gain \( K_d \). Then the feedback gain \( K_d \) is given by (11)

\[ K_d = K_o + 2 \sqrt{\frac{C_f}{L_f} \zeta} \]  
(11)

As a result, the stability of the system increases.
B. Input Power Factor Control

Fig. 8 shows two phasor vector diagrams when the input power factor is controlled at a lag and lead phase, respectively. It should be noted that the input power factor is decided by the $\theta_{pfc}$ as shown in Fig. 6. The input terminal voltage $V_i$ that is smaller than the induced e.m.f. $V_g$ is obtained in the case of lag power factor as shown in Fig. 8 (a). On the other hand, the input terminal voltage becomes larger than induced e.m.f. in the case of lead power factor as shown in Fig. 8 (b).

In these cases, an apparent power $S$ and an active power $P$ are obtained by (12) and (13)

$$S = V_i (I_i + jI_q)$$

$$P = V_i I_q$$

From (12) and (13), the input power factor $\cos \theta_{pfc}$ is calculated as shown in (14)

$$\cos \theta_{pfc} = \frac{P}{S} = \frac{V_i I_q}{\sqrt{I_i^2 + I_q^2}}$$

Next, the relation among the input terminal voltage, induced e.m.f. of the generator and the output voltage is considered.

A maximum voltage transfer ratio of the matrix converter is limited to 0.866 of the input terminal voltage. In addition, the voltage transfer ratio depends on the input power factor as shown (1). As a result, the maximum voltage $V_{out}$ is constrained as (15)

$$V_{out} = \lambda V_i \cos \theta \ (0 \leq \lambda \leq 1)$$

where $\lambda$ is a modulation index and $\cos \theta$ is the input power factor.

From (14) and (15), the output voltage $V_{out}$ is given by (16)

$$V_{out} = \lambda V_g \cos \theta_{pfc} = \lambda V_g \frac{I_q}{\sqrt{I_i^2 + I_q^2}}$$

In (16), $I_q$ is reactive current. In order to maximize generator efficiency, the input current is controlled as $I_q = 0$. Therefore, the maximum output voltage $V_{outmax}$ is obtained by (17)

$$V_{outmax} = \lambda V_g \frac{I_q}{\sqrt{I_i^2 + I_q^2}}$$

Fig. 9 shows a simulation result among the phase angle $\theta_{pfc}$, the generator terminal voltage and the output voltage. The maximum value of the output voltage is obtained at $\theta_{pfc} = 26$ deg. In contrast, the output voltage $V_{out}$ becomes smaller than the output voltage command in the case of lag power factor ($\theta_{pfc} < 0$ deg) and lead power factor ($\theta_{pfc} > 20$ deg). In addition, the generator terminal voltage increases in the case of lead power factor ($\theta_{pfc} > 0$ deg). As a result, the operation of the vector diagram in Fig. 8 is confirmed. Therefore, when a generator is used as the power supply, the input current phase should be controlled that the phase of the input current is decided by the induced e.m.f. in order to obtain the maximum voltage transfer ratio and high generator efficiency.

C. Output Voltage Compensation Control

The terminal voltage of the matrix converter is changed by the generator speed and current because the induced e.m.f of the generator depends on the generator speed and the voltage drop is caused by synchronous reactance. Therefore, in order to obtain the constant output voltage, the output voltage amplitude compensation is required.

The output voltage can not compensate by only the amplitude of the terminal voltage because the output voltage of the matrix converter depends on the input voltage and input power factor as described in (15). Generally, the input power factor of the matrix converter is always controlled to unity. Then, the output voltage is compensated according to the amplitude of the terminal voltage of the matrix converter. In case for the generator as input, the input power factor is changed by the load condition of the generator.

This paper proposes the output voltage compensation method using the frequency of the induced e.m.f. From (17), when a reactive input current $I_q$ is controlled to zero, the output voltage of the matrix converter is proportional to the induced e.m.f. Therefore, compensation is required in proportion to the induced e.m.f. The induced e.m.f can be detected from the generator speed. It should be noted that the generator speed is detected using a pole position sensor or estimated by the generator current and the input voltage.

Therefore a compensated output voltage command $V_{out}'$ is obtained by (18)

$$V_{out}' = \frac{f_{out}}{f_{in}} V_{out}$$

(18)
where $f_{\text{in}}$ is frequency of the generator terminal voltage, $f_{\text{rate}}$ is rated frequency of the generator and $v_{\text{out}}^*$ is the output voltage command.

The compensated output voltage command increases by the decrease of the generator speed.

Fig. 10 shows a system block diagram using the input stability control and proposed output voltage compensation method. Firstly, the pole position of the generator is detected by the pole position sensor. Secondly, the induced e.m.f. phase and generator frequency are calculated by digital Phase Locked Loop (PLL). The induced e.m.f. phase is used for not only the output voltage compensation but also the input power factor control.

The maximum value of the output voltage is limited to 0.866 of the induce e.m.f. even if the proposed compensation method is applied. When the output voltage command is over the 0.866, the harmonic distortion occurs on the output voltage.

IV. EXPERIMENTAL RESULTS

Fig. 11 shows the experimental results of a generator supply with a R-L load. The experimental parameters of Table 1 and 2 were used. Fig. 11 (a) shows waveforms of the input and output voltage and current with the conventional control method. Fig. 11 (b) shows waveforms using the proposed stability control method.

In the case of Fig. 11 (a), as mentioned in the simulation result in Fig. 3, large oscillation occurs in the input terminal voltage and input current. In addition, output waveforms have a distortion due to the resonance of the input side. As for Fig. 11 (b), the oscillation of the input side is drastically suppressed in comparison with Fig. 11 (a). Moreover, the distortion of the output side is eliminated. These results lead to the conclusion that the proposed stability control is extremely effective for a generator power supply.

Fig. 12 shows the experimental results that relation among the phase angle $\theta_{\text{foc}}$, the generator terminal voltage and the output voltage. As mentioned in the simulation result in Fig. 9, the output voltage $V_{\text{out}}$ becomes smaller than the output voltage command in the case of lag power factor ($\theta_{\text{foc}} = 0 \text{ deg}$). The maximum value of the output voltage is obtained at $\theta_{\text{foc}} = 26 \text{ deg}$. On the other hand, the output voltage decreases at $\theta_{\text{foc}} > 26 \text{ deg}$. However, the generator terminal voltage still rises. Therefore, when a generator is used as the power supply, the input current phase should be controlled at the same phase as the induced e.m.f. in order to obtain the maximum voltage transfer ratio.

Fig. 13 shows the experimental result of a generator – motor system. A general purpose induction motor of 1.5kW is used with V/f control. As mentioned in the experimental result in Fig. 11 (b), clean sinusoidal input current waveforms are obtained and total harmonic distortion (THD) for the input current is 3.7%. Moreover, the output current THD is 1.7%. The proposed stability control is also

Table 1. Generator parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>3.7kW</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>0.095Ω</td>
</tr>
<tr>
<td>Rated rotational frequency</td>
<td>1800 rpm</td>
</tr>
<tr>
<td>d-axis inductance</td>
<td>62mH</td>
</tr>
<tr>
<td>q-axis inductance</td>
<td>153mH</td>
</tr>
<tr>
<td>V_out* (%)</td>
<td>150 Vrms</td>
</tr>
<tr>
<td>Number of pole</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2. Simulation and experimental conditions.

| Output power (constant)    | 1.5kW  |
| Modulation index          | Q8     |
| Output power factor       | Q8     |
| Filter capacitor          | 6.0µF  |
| Load                      |        |
| Simulation                |        |
| Current source load       |        |
| RL load                   | 6.25Ω, 5mH |
| Motor load                | 1.5kW, 4 pole Induction motor |

Fig. 11. Experimental waveforms with RL load.

Fig. 12. Experimental waveforms with RL load.

Fig. 13. Experimental waveforms with an induction motor load.
Fig. 14 shows accelerating and decelerating results of the induction motor. The induction motor speed is changed from 150 r/min to 1500 r/min at 0.3 sec. After that, the induction motor decelerated from 1500 r/min to 150 r/min at 0.3 sec. In this system, the generator terminal voltage may increase instantly by the acceleration and deceleration. In Fig. 14, the input current rises by the acceleration and deceleration. However, the generator terminal voltage does not change instantly by the acceleration and deceleration. Thus, the input current distortion does not increase as well. Moreover, the output voltage and current can be controlled to stable condition. These results lead to the conclusion that the proposed stability control is effective for acceleration and deceleration of the induction motor.

Fig. 15 shows accelerating and decelerating result of the generator with an R-L load. The generator speed decreased from 1800 r/min to 1350 r/min at 0.5sec, after that, increased to 1800 r/min at 0.5sec. The modulation index of the matrix converter is 0.4. Fig. 15 (a) and (b) show waveforms of the input/output voltage and input/output current where it without or with output voltage compensation, respectively. If without compensation, the terminal and output voltage decreased while the generator speed decreased. As a result, the output side becomes unstable when the generator speed decreases greatly. On the other hand, with compensation method, the output voltage can be kept constant even if the terminal voltage decreases. In addition, both the output voltage and current and the input current are in stable. The proposed output voltage compensation method does not affect the input side.

V. CONCLUSION

This paper proposes a model and analysis based on the behavior of the matrix converter with a generator as input. The feature of the proposed methods is to clarify the cause of instability conduction by using an ideal mathematical model.

In order to solve the instability, this paper proposed the input current stability control on the rotating frame. In addition, the relation among the input current phase, the terminal voltage, output voltage and the induced e.m.f. of the generator was analyzed, and the optimal power factor control is proposed. Moreover, the output voltage compensation using generator speed was proposed.

The experimental and simulation results led to the conclusion that the matrix converter was valid for the generator power supply using the proposed stability control and the optimal power factor control. The proposed control method could achieved stability at motor load 1.5kW and the input current THD of 3.7%.

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